A Theoretical and Experimental Study of the Noise Behavior of Subharmonically Injection Locked Local Oscillators

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***Abstract*—A method for the noise characterization of optically controlled subharmonically injection locked oscillators is presented. Based on a nonlinear model of synchronized oscillators, this method is used to formulate a general expression for phase noise calculation, so that FM noise degradation of a subharmonically synchronized LO at large-signal levels can be predicted easily and accurately. The theoretical analysis shows that 1) the nth-order subharmonic injection locking oscillator is primarily locked by the nth harmonic output of an injected signal, which is generated by the nonlinearity of the active device; 2) the minimum FM noise degradation factor of the nth order subharmonically locked oscillator is n^2 when the injection power is sufficiently strong; 3) a subharmonic injection locking LO with low injection power, good FM noise degradation and large locking range can be designed by determining the optimum injection power level, by selecting the optimal nonlinear multiplication factor, and by decreasing the intrinsic noise level of the active device. The experimental results of the FM noise measurement of an oscillator confirmed the accuracy of the analysis.**

I. Introduction

THE NEXT generation of communication satellites will be based on large aperture phased array antennas, where each transmit/receive (T/R) module is interfaced to the central processor by a fiber-optic link, which distributes reference, data, and control signal [1]. Maintaining the phase and frequency stability of the reference signal for establishing a coherent carrier at each active T/R module is of great importance. To establish a coherent carrier, the frequency reference is used to synchronize local oscillators through indirect subharmonic optical injection locking techniques [2]. Because figures of merit, such as the locking range and the FM noise degradation of the subharmonically injection locked LO, have a great effect on the frequency stability and the noise behavior of the modulated carrier, it is important to analyze the subharmonic injection locking process quantitatively.

The analysis and the experimental verification of the large-signal model of the subharmonically injection locked oscillator has been reported recently by Zhang *et al.* [3], and the salient points of this analysis relevant to the noise calculation are presented here. In terms of noise behaviors, the noise theory for the fundamentally synchronized oscillator has been well studied by Kurokawa [4] in 1969, who used the Van der Pol nonlinear equivalent circuit model and the first order approximation to derive an elegant expression of noise characteristics at a low injection signal level. To characterize the synchronized oscillator noise at a large injection signal level, Goedbloed and Vlaardingerbroek [5], [7] developed a nonlinear noise theory for IMPATT diodes in oscillators and amplifiers. The method involves solving the nonlinear Read equation of IMPATT for large noise-free carrier signal first, then solving the linearized equation again for the small noise perturbation signal. With their theory, the noise behavior of free-running IMPATT oscillators and amplifiers can be analyzed with good accuracy.

Schunemann [6] reported another method larger-signal noise analysis that directly extends Kurokawa's work to the large injection level. This nonlinear model is based on a third order Van der Pol polynomial for a negative resistance oscillator. He used the describing function method to solve the large noise-free signal response of fundamentally injection locked oscillator. Like Goedbloed and Vlaardingerbroek, he also developed the level dependent conversion matrices for linear analysis of small noise signal on the basis of this analysis. However, all of above works concentrate only on the fundamentally synchronized oscillator, and to the best of our knowledge, no research work on the noise characteristics of the subharmonically injection locked LO has been published. Usually, a much higher injection power level is needed to obtain certain locking range for subharmonic injection locking than that required for fundamental injection locking. Thus, a means of analyzing the noise characteristics of the subharmonically injection locked oscillator at a large injection power level is needed.

The theory for the noise analysis of subharmonically injection locked LO's at large-signal levels is developed in this paper. In Section II, the nonlinear model for injection locked local oscillators is briefly introduced. Because the Van der Pol equivalent circuit model representation is not easily implemented for microwave oscillators [8], a more suitable nonlinear model [9], [10] for subharmonic

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injection locking was employed. Also, the order of nonlinearity used to represent a nonlinear device is extended to infinity to obtain a more general model for large injection signal levels. With this model, a noise free analysis is done for both the large oscillation signal and the injection signal via the harmonic balance method. In Section III, the method for noise characterization at large-signal level is described. The level dependent *conversion matrices* were developed for the small noise signal analysis via a linear algebraic operation. The noise conversion and the intermodulation effects on output noise could be calculated easily by use of these matrices. Therefore, not only the FM noise degradation of a subharmonic injection locking LO could be quantitatively analyzed, but also the AM noise response and the conversion between AM and FM noise could be characterized.

The theoretical results of FM noise characteristics at large injection signal levels were verified by noise measurements of a subharmonically locked oscillator at subharmonic factors of 1/2, 1/3, and 1/4, and are presented in Section IV. Thus, on the basis of this noise analysis, the optimum injection power level can be determined for a slave oscillator with small FM noise degradation. Furthermore, an optimum oscillator configuration can also be investigated for the optically controlled phased array antenna architectures.

II. Noise-Free Large Signal Analysis

A. Nonlinear Model for the Synchronized Oscillator

The nonlinear circuit of the oscillator, depicted in Fig. 1, is modeled as the combination of a pure nonlinear network $f(e)$ and a pure linear feedback network H(D) [9]. Here e_i is the injected signal, $\boldsymbol{u}$ is the output signal of the oscillator, and $\boldsymbol{e}_o$ is the signal from the feedback network. The input-output nonlinearity of the active device is represented by $f(e)$; e.g., $f(e)$ represents the current-voltage characteristics for a MESFET based device. To simplify the analysis, we express $f(e)$ approximated by

$$u = f(e) = \sum_{i=1}^{\infty} \alpha_i e^i \tag{2.1}$$

where α_i is considered to be real for simplicity. The linear single tuned feedback network $H(D)$ [11] can be expressed as approximately

$$H(D) = \frac{H_0}{1 + j2Q \dfrac{\Delta\omega}{\omega_0}} \tag{2.2}$$

where Q is the quality factor of this linear network, and $\Delta\omega$ is the frequency deviation from the resonating frequency ω_0 of the feedback circuit. Because we consider only the noise-free signal in this section, the output of this network, $\boldsymbol{e}_o$, is desired to be a sinusoidal signal. When the signal $\boldsymbol{e}_i$ is injected and the oscillator is locked, the input

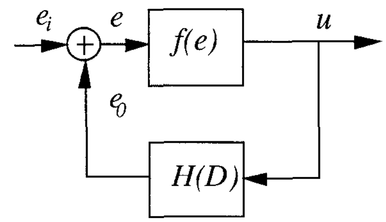

Fig. 1. The conceptual diagram of the subharmonically synchronized oscillator.

signal $\boldsymbol{e}$ for the nonlinear network is

$$e = e_0 + e_i = \frac{E}{2}(e^{j\omega t} + e^{-j\omega t}) + \frac{\dot{E}_i}{2} e^{j(\omega/n)t} + \frac{\dot{E}_i^*}{2} e^{-j(\omega/n)t} \tag{2.3}$$

where $\omega = n\omega_{\text{inj}}$ is the synchronized frequency by injection locking, and ω_{inj} is the injection frequency. The complex variable $\dot{E}_i$ is the injected signal, represented by the amplitude E_i and the phase ϕ; n is an integer for the subharmonic factor; E is the oscillation signal's amplitude at input port. Substituting (2.3) into (2.1), for the subharmonic injection at a factor of $1/n$, the output of oscillator can be expanded in a Fourier Series:

$$u = f(e) = \sum_{m=-\infty}^{\infty} \dot{U}_m e^{jm(\omega/n)t}. \tag{2.4}$$

Since $e_o = H(D)u$, finding the harmonic balance at the fundamental frequency ω will result in:

$$E = \frac{H_0}{1 + j2Q \dfrac{\Delta\beta}{\omega_0}} \dot{U}_n. \tag{2.5}$$

We can express U_n, which is the signal at the locked oscillation frequency $n\omega_{\text{inj}}$, in a summation form by following the method of the frequency domain nonlinear analysis with multiple inputs [12]:

$$\dot{U}_n = \left(\sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{2^{N-1}} \frac{N!}{(j!)^2(k+1)!k!} \alpha_N |E_i|^{2j} E^{2k}\right) E + \left(\sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \frac{1}{2^{M-1}} \frac{M!}{(m!)^2(p+n)!p!} \alpha_M |E_i|^{2p} E^{2m}\right) \cdot \dot{E}_i^n + \text{higher order terms}. \tag{2.6}$$

where $N = 2j + 2k + 1$ and $M = 2m + 2p + n$. In (2.6), the first term is the oscillation signal amplitude U_{out}; and the second term represents the response signal U_{outn} of the injected signal E_i, when E_i goes through the nonlinear network together with the oscillation signal E. The above equation can be simplified as:

$$\dot{U}_n \approx U_{\text{out}} + \dot{U}_{\text{outn}} \tag{2.7}$$

Substituting (2.7) into (2.5) produces the following expressions:

$$E = H_0 U_{\text{out}} \tag{2.8}$$

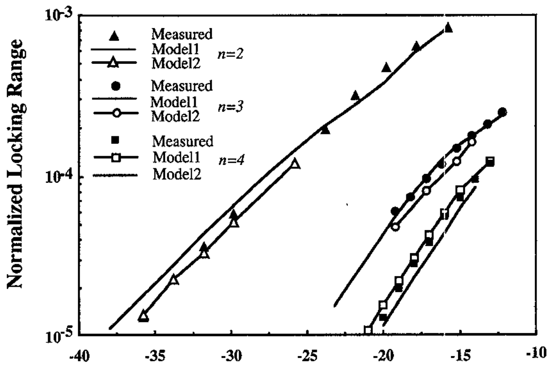

Fig. 2. Measured and calculated injection locking range ($\Delta f/f_0$) at subharmonic factor $n = 2, 3, 4$, where calculation for Modell represents the calculated results with Eq. (2.10), and calculation for Model2 represents the predicted results with unlocked spectrum [12].

$$\Delta\omega = \frac{\omega_0}{2Q}\frac{\mathrm{Im}\,(\dot{U}_n)}{\mathrm{Re}\,(\dot{U}_n)} \approx \frac{\omega_0}{2Q}\frac{\mathrm{Sin}\,(n\phi)\,|\dot{U}_{\mathrm{outn}}|}{U_{\mathrm{out}}} \tag{2.9}$$

Following the method of Daryoush [9], the subharmonic injection locking range can be expressed in terms of Q and ω_0 by letting $n\phi = \pm\pi/2$:

$$\Delta\omega_{1/n} \approx \frac{\omega_0}{2Q}\frac{U_{\mathrm{outn}}}{U_{\mathrm{out}}} = \frac{\omega_0}{2Q}\sqrt{\frac{P_{\mathrm{outn}}}{P_{\mathrm{out}}}} \tag{2.10}$$

Clearly, the above equation is the same as Alder's expression [13] for the fundamental injection locking range, when the signal U_{outn} interacts with the free-running oscillator like an injected fundamental locking signal.

B. Experimental Verification of Large-Signal Analysis

A 5 GHz local oscillator, designed and fabricated for the examination of the subharmonic locking range and noise characteristics, consists of a two-stage low-noise MESFET amplifier and a dielectric resonator as linear feedback network. The basic advantages of such an oscillator in a synchronized oscillator are outlined in Berceli *et al.* [10]. (2.10) and Armand/Strover's method [14], [15] were used to predict subharmonic locking ranges at subharmonic factors of 1/2, 1/3, and 1/4, and the results are discussed in more detail in Zhang [3]. Fig. 2 shows the comparison between the analysis and the measurement results.

This nonlinear model and its large-signal analysis for the subharmonic injection locking range calculation are verified by the locking range measurement presented in Fig. 2. Therefore, the subharmonic injection locking process can be explained by (2.10) as follows: the injection signal goes through the nonlinear network first, then its nth harmonic signal is generated at the output port of the nonlinear network. If the injection frequency at $n\omega_{\mathrm{inj}} \approx \omega_0$, the multiplied signal can be fed back to the nonlinear network again through the linear tank circuit. This signal forces the free-running oscillator to be synchronized at $\omega = n\omega_{\mathrm{inj}}$, the same as that in a fundamental locking. This process was also used to explain the noise behaviors of the subharmonically locked oscillator in the next section.

III. Noise Analysis

The rigorous noise analysis presented here is restricted to the noise calculation at the center of the locking range. As a frequency detuning exists between the free-running oscillation frequency and the injection signal, a phase shift of $\pm 90°$ can be introduced [16]. Although this assumption may seem restrictive, the phase error caused by shifting away from the center of the locking range can be remedied via the injection locked PLL technique [17]. The noise signal can be regarded as a small perturbation of a large oscillation and the injection signal, so that the total input signal of an active device e will be: $e = e_L + e_N$, where e_L is the large pure sinusoidal signal consisting of oscillating and injecting signals, and e_N is a small noise signal. Because $e_N << e_L$, the noise response of the nonlinear network $f(e)$ can be approximately expressed by the derivative of $f(e)$ in a linear form:

$$u_n \approx f'(e)\,|_{e=e_L} e_N \tag{3.1}$$

The expression for e_L is shown in (2.3). Substituting e_L into $f(e)$, we can express the derivative of $f(e)$ in a Fourier series shown below:

$$f'(e_L(t)) = \sum_{m=-\infty}^{\infty} g_m e^{jm(\omega/n)t} \tag{3.2}$$

$$g_m = \frac{1}{2\pi}\int_{-\pi}^{\pi} f'(e_L(t))\,e^{-jm(\omega/n)t}\,d\left(\frac{\omega}{n}t\right). \tag{3.3}$$

The noise signal can be considered as pairs of pseudosinusoids with random amplitudes and phases at the lower and upper side-band frequencies. We use e_{il} at $(\omega/n) - \Omega$ and e_{iu} at $(\omega/n) + \Omega$ to denote noise from the injection signal; e_l at $\omega - \Omega$ and e_u at $\omega + \Omega$ represent the feedback fundamental noise, and e_{ol}, e_{ou} denote the intrinsic noise side-band signals at the fundamental. The offset carrier frequency of side-band noise signals is Ω. Therefore, as shown in Fig. 3, there are noise side-band signals at the input port of the amplifier, presented as

$$e_{il}e^{j(\omega/n-\Omega)t};\quad e_{iu}e^{j(\omega/n+\Omega)t};\quad e_l e^{j(\omega-\Omega)t};$$
$$e_u e^{j(\omega+\Omega)t};\quad e_{ol}e^{j(\omega-\Omega)t};\quad e_{ou}e^{j(\omega+\Omega)t} \tag{3.4}$$

Similarly, Fig. 3 also shows that at the output port, noise components at the fundamental frequency band are

$$u_l e^{j(\omega-\Omega)};\quad u_u e^{j(\omega+\Omega)t} \tag{3.5}$$

Following (3.1), we can express the output noise signal at the fundamental frequency band by input signals in matrix form:

$$\begin{bmatrix} u_l^* \\ u_u \end{bmatrix} = \begin{bmatrix} g_0^* & g_{2n}^* \\ g_{2n} & g_0 \end{bmatrix}\begin{bmatrix} e_l^* + e_{ol}^* \\ e_u + e_{ou} \end{bmatrix} + \begin{bmatrix} g_{n-1}^* & g_{n+1}^* \\ g_{n+1} & g_{n-1} \end{bmatrix}\begin{bmatrix} e_{il}^* \\ e_{iu} \end{bmatrix}. \tag{3.6}$$

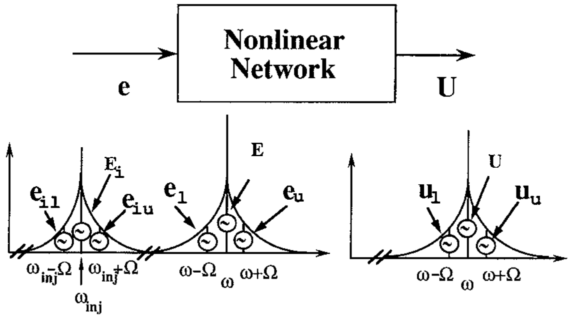

Fig. 3. Noise spectrum of input and output of nonlinear network expressed in terms of an equivalent spectrum of sinusoidal noise signals.

Through the transfer function of the feedback network in (2.2), the feedback noise signal can be related to the output noise signal:

$$\begin{bmatrix} e_l^* \\ e_u \end{bmatrix} = \begin{pmatrix} H_0\left(1 - j2Q\dfrac{(\omega - \Omega) - \omega_0}{\omega_0}\right)^{-1} & 0 \\ 0 & H_0\left(1 + j2Q\dfrac{(\omega + \Omega) - \omega_0}{\omega_0}\right)^{-1} \end{pmatrix} \begin{bmatrix} u_l^* \\ u_u \end{bmatrix}. \quad (3.7)$$

We can rewrite (3.6) and (3.7) in vector algebraic form respectively:

$$\boldsymbol{u} = G_0(\boldsymbol{e} + \boldsymbol{e}_0) + \boldsymbol{G}_n \boldsymbol{e}_i \quad (3.8)$$

where

$$\boldsymbol{u} = \begin{bmatrix} u_l^* \\ u_u \end{bmatrix}; \quad \boldsymbol{e}_x = \begin{bmatrix} e_{xl}^* \\ e_{xu} \end{bmatrix};$$

$$\boldsymbol{G}_0 = \begin{bmatrix} g_0^* & g_{2n}^* \\ g_{2n} & g_0 \end{bmatrix}; \quad \boldsymbol{G}_n = \begin{bmatrix} g_{n-1}^* & g_{n+1}^* \\ g_{n+1} & g_{n-1} \end{bmatrix}$$

and

$$\boldsymbol{e} = \boldsymbol{H}\boldsymbol{u} \quad (3.9)$$

where

$$\boldsymbol{H} = \begin{pmatrix} H_0\left(1 - j2Q\dfrac{(\omega - \Omega) - \omega_0}{\omega_0}\right)^{-1} & 0 \\ 0 & H_0\left(1 + j2Q\dfrac{(\omega + \Omega) - \omega_0}{\omega_0}\right)^{-1} \end{pmatrix}.$$

Here, $\boldsymbol{G}_i$ is referred to as the *conversion matrix* because it can convert the side-band signal from one frequency band to another.

Substituting (3.9) into (3.8), we can show the relationship between the injected noise and output noise of an oscillator as

$$\boldsymbol{u} = [\boldsymbol{I} - \boldsymbol{G}_0\boldsymbol{H}]^{-1}[\boldsymbol{G}_0\bar{\boldsymbol{e}} + \boldsymbol{G}_n\boldsymbol{e}_i] = \boldsymbol{G}_0'\boldsymbol{e}_0 + \boldsymbol{G}_n'\boldsymbol{e}_i \quad (3.10)$$

where $\boldsymbol{G}_0' = [\boldsymbol{I} - \boldsymbol{G}_0\boldsymbol{H}]^{-1}\boldsymbol{G}_0$ and $\boldsymbol{G}_n' = [\boldsymbol{I} - \boldsymbol{G}_0\boldsymbol{H}]^{-1}\boldsymbol{G}_n$. Usually, the oscillator noise is described in terms of FM and AM noise, so the two side-band noise signals have to be transformed into AM and FM noise in terms of vector algebra as [5]:

$$\boldsymbol{u}_N = \begin{bmatrix} u_{AM} \\ u_{FM} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_l \\ u_u \end{bmatrix} = \boldsymbol{A}\boldsymbol{u} \quad (3.11)$$

$$\boldsymbol{e}_N = \begin{bmatrix} e_{AM} \\ e_{FM} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e_{il} \\ e_{iu} \end{bmatrix} = \boldsymbol{A}\boldsymbol{e}_i \quad (3.12)$$

The measurable output AM and FM noise power level can be given by using these expressions:

$$\langle \boldsymbol{u}_N, \boldsymbol{u}_N^+ \rangle = \begin{bmatrix} \overline{|u_{AM}|^2} & \overline{u_{AM}u_{FM}^*} \\ \overline{u_{AM}^*u_{FM}} & \overline{|u_{FM}|^2} \end{bmatrix}$$
$$= \tfrac{1}{2}\boldsymbol{A}\boldsymbol{G}_0' \begin{bmatrix} \overline{|e_0|^2} & 0 \\ 0 & \overline{|e_0|^2} \end{bmatrix} \boldsymbol{G}_0'^{+}\boldsymbol{A}^{+} + \boldsymbol{A}\boldsymbol{G}_s'\boldsymbol{A}^{-1}$$
$$\cdot \begin{bmatrix} \overline{|e_{iAM}|^2} & 0 \\ 0 & \overline{|e_{iFM}|^2} \end{bmatrix} \boldsymbol{A}^{-1+}\boldsymbol{G}_s'^{+}\boldsymbol{A}^{+} \quad (3.13)$$

Here, we assume no correlation between input and intrinsic noise. The superscript '+' denotes the Hermitian matrix; $\overline{|e_0|^2}$ is the equivalent noise level of amplifier; $\overline{|e_{iAM}|^2}$ and $\overline{|e_{iFM}|^2}$ are the AM noise and FM noise level of the signal from master oscillator. Using the expression in (2.1) for nonlinear network and following the same rules of Bussagang *et al.* [12] in (2.6), we can express the elements in $\boldsymbol{G}_i$ in terms of nonlinear parameter as

$$g_0 = \Gamma_0 + \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{N!\Gamma_N|E_i|^{2j}E^{2k}}{2^N(j!)^2(k!)^2}$$
$$+ \sum_{m=0}^{\infty}\sum_{p=0}^{\infty} \frac{M!\Gamma_M E_i^{*n}|E_i|^{2p}E^{2m+1}}{2^M m!(m+1)!(p+n)!p!}$$
$$+ \text{higher order terms} \quad (3.14)$$

$$g_{n-1} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{N!\Gamma_N \dot{E}_i^{n-1} |\dot{E}_i|^{2j} E^{2k}}{2^N (j+n-1)! j! (k!)^2} + \sum_{m=0}^{\infty}\sum_{p=0}^{\infty} \frac{M!\Gamma_M \dot{E}_i^{*} |\dot{E}_i|^{2p} E^{2m+1}}{2^M m! (m+1)! (p+1)! p!} + \text{higher order terms} \quad (3.15)$$

$$g_{n+1} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{N!\Gamma_N \dot{E}_i^{n+1} |\dot{E}_i|^{2j} E^{2k}}{2^N (j+n+1)! j! (k!)^2} + \sum_{m=0}^{\infty}\sum_{p=0}^{\infty} \frac{M!\Gamma_M \dot{E}_i |\dot{E}_i|^{2p} E^{2m+1}}{2^M m! (m+1)! (p+1)! p!} + \text{higher order terms} \quad (3.16)$$

$$g_{2n} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{N!\Gamma_N |\dot{E}_i|^{2j} E^{2k+2}}{2^N (j!)^2 (k+2)! k!} + \sum_{m=0}^{\infty}\sum_{p=0}^{\infty} \frac{M!\Gamma_M \dot{E}_i^{n} |\dot{E}_i|^{2p} E^{2m+1}}{2^M m! (m+1)! (p+n)! p!} + \text{higher order terms} \quad (3.17)$$

where $\Gamma_i = (i+1)\alpha_{i+1}$ in the above expressions, and M and N are equal to the sum of the exponents of E and Ei. For example, $N = 2j + 2k$ and $M = 2m + 2p + n + 1$ in (3.14). Those higher order terms are at least n order higher than the first and the second terms in above equations, therefore, we can neglect the contribution from these higher order terms. If the injected AM noise level is much smaller than the FM noise level and the synchronized frequency is in the center of the locking range, then g_{ij} is a real number because $\phi = 0$, and the following expression for the output FM noise power level can be derived from the noise matrix in (3.13):

$$\overline{|u_{FM}|^2} = \frac{(g_{n-1} - g_{n+1})^2 \overline{|e_{FM}|^2} + \frac{1}{2}(g_0 - g_{2n})^2 e_0^2}{(1 - (g_0 - g_{2n}) H_0)^2 + (2Q\Omega/\omega_0)^2}. \quad (3.18)$$

Substituting (3.14)–(3.17), (2.8) and (2.10) into (3.18), a simple expression for the output FM Noise/Carrier ratio $£_{1/n}(\Omega)$ of a nth order subharmonically locked LO can be formulated in terms of the injection FM Noise/Carrier ratio $£_{inj}(\Omega)$ and the intrinsic noise e_o:

$$£_{1/n}(\Omega) = \frac{\Delta\omega_{1/n}^2 n^2 £_{inj}(\Omega) + \left(\frac{\omega_0}{2Qb}\right)^2 \frac{P_N}{P_{out}}}{\Omega^2 + \Delta\omega_{1/n}^2} \quad (3.19)$$

where P_N is the equivalent intrinsic noise power density at the input port, P_{out} is the output power of the oscillator, and b is the coupling factor of the feedback network [10]. Clearly, the FM noise degradation factor will be $\boldsymbol{n}^2$ when the injection locking range is large enough. This result corresponds to the empirically observed relation of 20 $\text{Log}_{10}(\boldsymbol{n})$ for the FM noise degradation in the logarithm [1]. The second term in the numerator of (3.19) is the variance of the free-running oscillation frequency caused by the noise power in 1 Hz bandwidth. Therefore, we use a $\Delta\Omega$ to represent this frequency jittering:

$$\Delta\Omega = \frac{\omega_0}{2Qb}\sqrt{\frac{P_N}{P_{out}}} \quad (3.20)$$

We may notice that the above equation is similar to the expression for the injection locking range calculation; thus, it can be considered as an average of the frequency shift by the average noise power within 1 Hz through the locking process.

While the synchronized frequency is not in the center of locking range, namely, $\phi \neq 0$, the following expression for the subharmonically locked LO FM noise can be deduced by omitting the second term in (3.14) and (3.17) for an input power that is not too large:

$$£_{1/n}(\Omega) = \frac{\cos^2(n\phi)\,\Delta\omega_{1/n}^2 n^2 £_{inj}(\Omega) + \Delta\Omega^2}{\Omega^2 + \cos^2(n\phi)\,\Delta\omega_{1/n}^2}. \quad (3.21)$$

This deduction may not be very rigorous, but for an injection signal that is not too large, (3.21) compares well with the empirical method used in [16]. Clearly, for the fundamental injection locking ($n = 1$), the above expression is the same as the FM noise expression for the fundamental synchronization in [4]. For subharmonic injection locking, injected FM noise converted by the nonlinear network to FM noise in the fundamental band suffers a degradation factor of n^2. This converted noise then reacts with the free-running oscillator like an injected fundamental FM noise similar to that shown in (2.10) for a subharmonic injection locking range calculation, and this proves the explanation for subharmonic injection locking stated in last section.

Taking into account the near-carrier noise of a MESFET based oscillator, we can simply replace e_o^2 in (3.19) and (3.21) with $(1 + \omega_c/\Omega)e_o^2$ [4], [18]

$$£_{1/n}(\Omega) = \frac{\Delta\omega_{1/n}^2 n^2 £_{inj}(\Omega) + \left(1 + \frac{\omega_c}{\Omega}\right)\Delta\Omega^2}{\Omega^2 + \Delta\omega_{1/n}^2} \quad (3.22)$$

where ω_c is the corner frequency for $1/f$ near-carrier noise of the device.

IV. FM Noise Experimental Verification and Discussion

The DR oscillator at 5 GHz [3] with an output power of 13 dBm was used for the FM noise measurement. A HP8340B synthesizer was used to provide injection signal, and the FM noise was monitored by the Tektronix 2756p spectrum analyzer.

First, the phase noise of the free-running oscillator was measured at different offset carrier frequencies Ω. Using the formula for the free-running oscillator derived from (3.22), we calculated the corner frequency f_c and the fac-

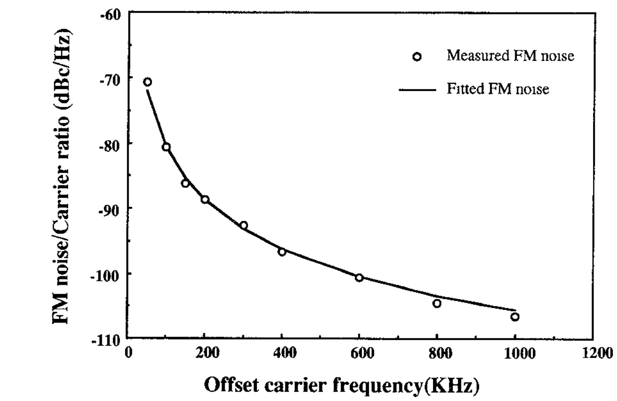

Fig. 4. Curve fitting of free-running oscillator phase noise, where the fitted results are: corner frequency $fc = 337$ kHz; and the factor for intrinsic noise $\Delta\Omega^2 = 20/\text{Hz}$.

tor $\Delta\Omega$ by curve fitting [18]:

$$£_0(\Omega) = \left(1 + \frac{\omega_c}{\Omega}\right)\frac{\Delta\Omega^2}{\Omega^2} \tag{4.1}$$

Fig. 4 shows the results of the FM noise curve fitting. Then we measured the phase noise of injection signal at different Ω and subharmonic factors. Before it was injected into oscillator, the FM noise coming from HP8340B was characterized by a Tektronix 2756p spectrum analyzer. It should be mentioned that the injected close-in carrier FM noise can not be measured accurately for 1/3 and 1/4 subharmonic injection locking because of very low phase noise levels at such frequencies (i.e. 1.25 GHz and 1.667 GHz). Therefore, an indirect method was used to measure the phase from the master generator. In this method, the injected signal was multiplied in frequency and phase noises at the 2nd, 3rd, and 4th harmonic frequencies were measured; the phase noise of the multiplied signal was divided by a factor of the square of the harmonic order to calculate the fundamental phase noise level [19].

After the characterization of the FM noise of the generator and the free-running oscillator, the oscillator was subharmonically injection locked at factors of 1/2, 1/3, and 1/4. The locking ranges and phase noises at the offset frequencies of 1 kHz, 10 kHz, 50 kHz and 100 kHz were measured at different input power levels. Fig. 5(a)–(c) shows comparisons between the measured and calculated phase noises which vary with injected signal power at subharmonic factors of 1/2, 1/3, and 1/4, respectively. Fig. 6 shows comparisons of the measured and calculated FM noise levels with (3.21) as a function of the detuning frequency, $\Delta f = nf_{\text{inj}} - f_o$. In this case, the subharmonic factor is 1/4, the input power is -3 dBm. Figs. 5 and 6 show that the measured and analyzed results were in good agreement.

Fig. 5 shows that the FM noise may decrease with the increasing injection power level and the noise will almost remain constant after a certain level, and the minimum output FM noise is approximately higher than the injected FM noise by a factor of n^2. To explain these phenomena,

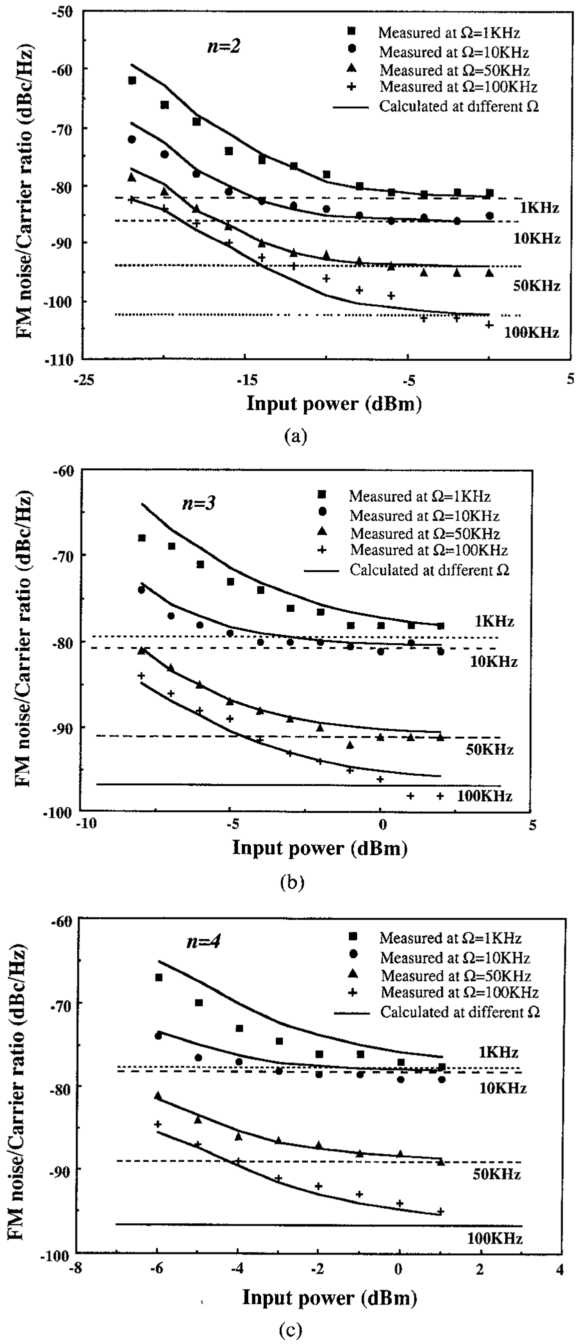

Fig. 5. Comparison of measured and calculated phase noise at offset carrier frequencies of 1 kHz, 10 kHz, 50 kHz, and 100 kHz, as a function of injection signal power. (a) Phase noise of the 2nd order subharmonic injection oscillator. (b) Phase noise of the 3rd order subharmonic injection oscillator. (c) Phase noise of the 3rd order subharmonic injection oscillator. The dashed straight-lines represent values of $n^2£_{\text{inj}}(\Omega)$.

we can, considering the close-in carrier phase where $\Omega << \Delta\omega_{1/n}$, simplify (3.21) as

$$£_{1/n}(\Omega) \approx n^2£_{\text{inj}}(\Omega) + \frac{\left(1 + \frac{\omega_c}{\Omega}\right)}{\Delta\omega_{1/n}^2}\Delta\Omega^2 \tag{4.2}$$

When $\Delta\omega_{1/n}$ is small, the second term in (4.2), namely the contribution of the intrinsic noise, is dominant. Then the relation of the output FM noise with the locking range

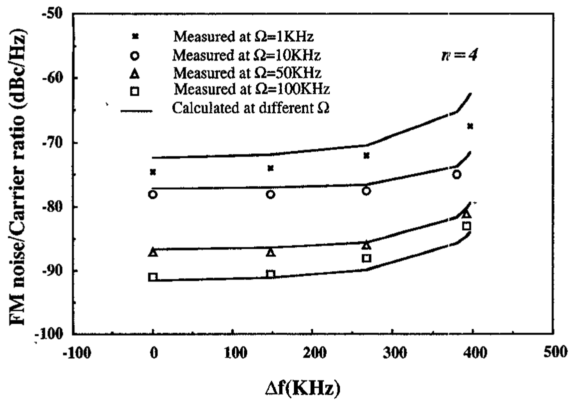

Fig. 6. Measured and calculated phase noise of injection locked oscillator at subharmonic factor of 1/4, as a function of $\Delta f = nf_{inj} - f_o$.

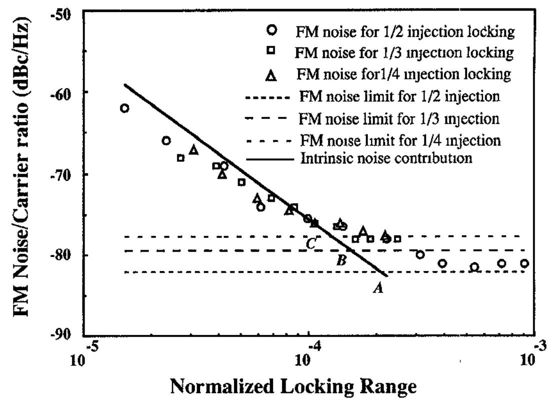

Fig. 7. Measured and calculated phase noise at 1 kHz offset carrier frequency of the injection locked oscillator at subharmonic factor of 1/2, 1/3 and 1/4, as a function of the injection locking range.

is about $1/x^2$. Thus, when the input power increases, $\Delta\omega_{1/n}$ increases and the output FM noise decreases. This phenomenon can be seen in Fig. 7, which shows the FM noise at the 1 kHz offset carrier varying with the injection locking range at subharmonic orders $n = 2$, 3, and 4, where a solid line represents the true $1/x^2$ relation. When the input power reaches certain level, $\Delta\omega_{1/n}$ becomes large enough so that the contribution of the second term is very small and makes $£_{1/n}(\Omega)$ approach to a minimum valued $n^2£_{inj}(\Omega)$. This is shown in Fig. 7 as FM noises diverge from the line of $1/x^2$, and slowly approach the dashed lines. These lines represent the first term in (4.2) at different subharmonic factor, namely, the minimum limits of FM noise. Therefore, we can conclude that the minimum FM noise degradation factor is n^2 for an nth order subharmonic injection locking oscillator.

In Fig. 7, those intersections A, B, and C between the line of $1/x^2$ and the limit lines represent turning points where the two terms in (4.2) are equal to each other. Referred to as 3 dB turning points, these points indicate how much locking range or injection power is needed to obtain the FM noise level which is 3 dB higher than minimum level. As discussed above, when the locking range is smaller than the 3 dB turning point, the FM noise degradation can be improved by an order of square via increasing the injection locking range; when it gets larger than this point, FM noise will be improved very slowly via increasing injection locking range and only 3 dB to be improved. Therefore, by taking advantage of this 3 dB turning point, we can determine the optimum locking range or injection power to obtain an optimum FM noise degradation. The locking range for 3 dB point can be calculated simply through this expression:

$$\Delta\omega_{1/n} \approx \frac{\sqrt{1 + \omega_c/\Omega}}{\sqrt[n]{£_{inj}(\Omega)}} \Delta\Omega. \tag{4.3}$$

The expression shows that to reach the 3 dB turning point of FM noise degradation, a larger locking range is required for a higher intrinsic noise level P_N or a lower injection phase noise. Substituting (2.10) and (3.20), we can express the injection power needed to reach the 3 dB turning point as

$$P_{inj} \approx \frac{1}{b^2n^2}\left(1 + \frac{\omega_c}{\Omega}\right)\frac{P_N}{£_{inj}(\Omega)M} \tag{4.4}$$

where M is the nonlinear network multiplication factor for the subharmonic injection signal. Clearly, if the multiplication factor is high, the injection power needed to reach the 3 dB turning point will be small, and the locking range will also be large which can seen from (2.10).

V. Conclusion

A method for the accurate analysis of the noise behavior of subharmonic injection locked LO's is presented in this paper. A general expression for output FM noise calculation was formulated in terms of injected phase noise, intrinsic noise of the free-running oscillator and the injection locking range. The theoretical model was verified through the FM noise measurement of a 5 GHz DRO. The analyzed and measured results both showed that 1) the minimum FM noise degradation factor of an nth order subharmonically locked oscillator is n^2; 2) FM noise degradation can only be improved efficiently by increasing input power before it reaches 3 dB point. These results formed the groundwork for a successful design of a subharmonic injection locking oscillator with the following requirements: large locking range, minimal noise degradation, minimum injection power level. We feel that these requirements can be satisfied by proper design of a circuit topology where a high multiplication factor with a low 3 dB noise degradation point can be achieved.

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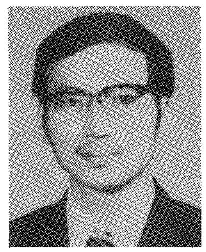

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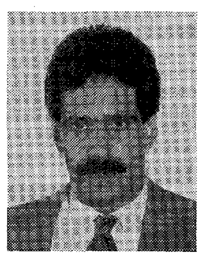

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